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Scaling Of Specimen Boundary Effect On Quasi-Brittle Fracture

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ABSTRACT: The size effect on quasi-brittle fracture is modelled by considering the distance of crack-tip to specimen boundary. It is shown that the tensile strength criterion applies if specimen boundary is very close to the crack-tip, and the fracture toughness criterion applies if specimen boundary is far away from the crack-tip. The specimen boundary influence captures the mechanism of size effect, i.e. the interaction of the crack-tip fracture process zone with specimen boundary. The boundary effect model proposed in the study is then compared with the common size effect model, which emphasizes the specimen size influence, by analysing the same experimental results.

1 INTRODUCTION

Size effect on material fracture behaviour has been studied extensively by both macro- and micro-mechanics [Bažant, 1984; Carpinteri and Chiaia, 1995; Karihaloo et al, 2003]. For instance, in the field of macro-mechanics, size effect is well-known for concrete specimens commonly measured from 100 mm to 5,000 mm in size [e.g. Karihaloo et al, 2003]. Although the absolute specimen size is huge, the concrete structure (typically with maximum aggregate size above 5 mm) and specimen size ratio is very similar to that of some micro-specimens used for advanced material systems such as thin films. For example, micro-specimens of polysilicon measured from 2.5 to 7.5 μm in thickness and from 6 to 20 μm for the uncracked ligament have been used to determine the fracture toughness [Ballarini et al, 1997]. The average grain size polysilicon is typically around 200 nm or larger. Therefore, those macro-concrete and micro-polysilicon specimens have an almost identical material structure and specimen size ratio.

Despite the huge difference in specimen size, those macro- and micro specimens may face similar size effect issues because they have similar material structure and specimen size ratios. Although the current study is confined mainly to macro- fracture mechanics, some size issues relevant to micro-specimens can also be addressed so that communication between the macro- and micro-mechanics modelling can be established. Therefore, the common size effect on quasi-brittle fracture is studied with special attention to its fundamental mechanism, i.e. the physical origin of the apparent specimen size effect.

2 MODELLING OF QUASI-BRITTLE FRACTURE

Arguably, the most well-known size effect model on quasi-brittle fracture of concrete-like materials was proposed by Bazant [1984]. The nominal strength, σ_N, of a specimen with an initial notch was given by:

$$\sigma_N = \frac{A \cdot \sigma_T}{\sqrt{1 + W/W^*}} \tag{1}$$

where σ_T is the tensile strength, A and W^* are two scaling parameters, and W is the specimen size. Equation (1) shows that there are two asymptotic limits for quasi-brittle fracture of concrete-like materials. The strength criterion σ_T is the dominant criterion for very small specimen size W while the fracture toughness criterion K_{IC} is the dominant criterion for very large W. The scaling parameters A and W^* need to be determined from experimental results through curve-fitting. The curve-fitting process requires geometrically similar specimens so that A and W^* become constant.

It should be pointed out that under the condition of geometric similarity, the α-ratio (= a/W or the crack-size/specimen-size ratio) also remains constant. If specimen size W is small, crack size a is small, and if W is large, so is a. It seems the influence of crack size on quasi-brittle fracture has been mingled into the size effect relation in equation (1) by the condition of geometric similarity. It is conceivable that a suitable specimen size can be found for a given material so that K_{IC} criterion applies for a moderate α-ratio, e.g. 0.5. However, fracture of specimens of the same size can still be quasi-brittle if the α-ratio is either close to zero or close to one, which indicates the distance of crack-tip to specimen boundary also contributes to the apparent specimen size effect. The distance of the crack-tip to specimen boundary has to be considered when the crack-tip fracture process zone (FPZ) is comparable in size.

To emphasize the influence of specimen boundary, an idealised specimen condition, a large plate with a small edge crack (the geometry factor Y = 1.12), has been selected. It is assumed that the specimen size W is big enough so that it does not need to be modelled. In this case, quasi-brittle fracture of the large plate is given by [Hu, 1998, 2002; Hu and Wittmann, 2000]:

$$\sigma_N = \frac{\sigma_T}{\sqrt{1 + a/a^*_\infty}} \tag{2}$$

where the reference a^*_∞ is a measurement of the crack-tip FPZ for a quasi-brittle material or the crack-tip plastic zone for a ductile material, and is equal to $0.25\cdot(K_{IC}/\sigma_T)^2$. Like equation (1), equation (2) has two well-defined asymptotic limits, σ_T and K_{IC}, for a very short and very long crack, respectively. The non-linear elastic fracture problems described by equation (2) can also be found in other material systems. For instance, the traditional elastic and plastic fracture of metals has the same asymptotic limits with σ_T and K_{IC} as the two extreme failure criteria.

Equation (2) describes the interactions between FPZ and specimen front face boundary since only a small edge crack is considered for the large plate. Commonly used fracture mechanics specimens do not satisfy this condition. This is because the specimen back face boundary may also be fairly close to FPZ depending on the length of uncracked ligament (W-a). The geometry factor Y = $Y(\alpha)$ also varies with α-ratio.

Linear elastic fracture mechanics (LEFM) may still be applicable even if the large plate condition Y = 1.12 is not satisfied (for instance, a concrete specimen with W = 1 m and α-ratio of 0.5). The nominal strength σ_N (which does not consider the presence of a crack) and fracture toughness K_{IC} are then related.

$$K_{IC} = \sigma_N \cdot Y(\alpha) \cdot \sqrt{\pi a} \tag{3}$$

For a finite-sized specimen, the strength criterion σ_T is expected to dominate when α-ratio is close to 0 or 1 [Duan and Hu, 2002, 2004a,b; Duan et al, 2004].

Figure 1 shows another nominal strength σ_n (which considers the presence of a crack but not stress singularity) [Duan and Hu, 2004a,b; Duan et al, 2004] together with σ_N. For the three-point-bending (3-p-b) situation, it can be found,

$$\begin{aligned} \sigma_N &= \mathrm{A}(\alpha) \cdot \sigma_n \\ \mathrm{A}(\alpha) &= (1-\alpha)^2 \end{aligned} \tag{4}$$

The $\mathrm{A}(\alpha)$ can also be easily determined for other specimen geometry such as compact tension or single-edge-notch-tension, following the definition shown in Figure 1.

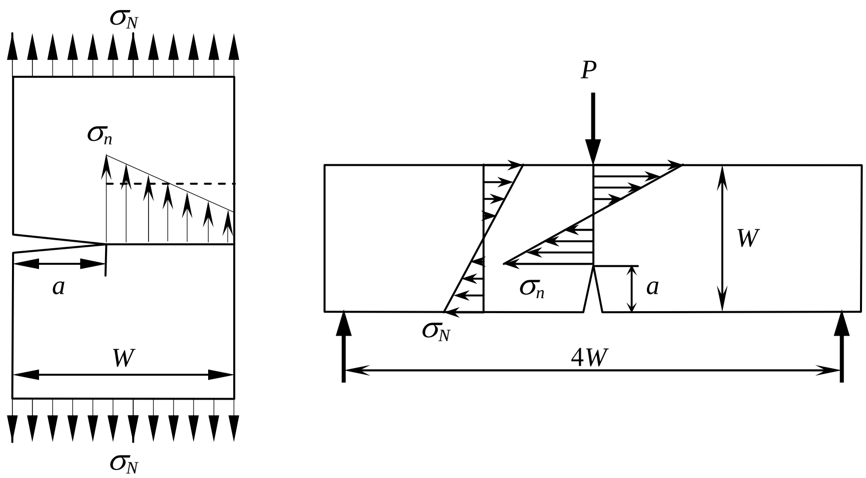

Figure 1. The single edge notched tension (SENT) and 3-point-bending (3-p-b) specimens with two nominal strengths defined, σ_N without consideration of the crack and σ_n with consideration of the crack.

Both σ_N and σ_n can be easily determined from the maximum load. If $\alpha \approx 0$ (such as the large plate case), the two nominal strengths, σ_N and σ_n, are identical, and both should approach σ_T if the strength criterion is applicable. If $\alpha \approx 1$, it is expected that the strength criterion σ_T should again be applicable, but only σ_n can be used to compare with σ_T as $\sigma_N \approx 0$. It is clear that to cover the entire α-ratio from 0 to 1, σ_n should be used instead of σ_N.

The LEFM situation described by equation (3) can be taken as the asymptotic solution when the fracture toughness K_{IC} criterion is valid. σ_n can be solved from equations (3) and (4) as follows [Duan and Hu, 2002, 2004a,b; Duan et al, 2004],

$$\sigma_n = \frac{K_{IC}}{\mathrm{A}(\alpha)\cdot Y(\alpha)\cdot\sqrt{\pi a}} = \frac{\sigma_T}{\sqrt{\dfrac{\left(\dfrac{\mathrm{A}(\alpha)\cdot Y(\alpha)}{1.12}\right)^2\cdot a}{\dfrac{1}{1.12^2\pi}\cdot\left(\dfrac{K_{IC}}{\sigma_T}\right)^2}}} = \frac{\sigma_T}{\sqrt{\dfrac{a_e}{a_\infty^*}}} \tag{5}$$

The equivalent crack a_e and the reference crack a^*_∞ first introduced in equation (2) are given by:

$$a_e = \left(\frac{A(\alpha)\cdot Y(\alpha)}{1.12}\right)^2\cdot a \tag{6}$$

$$a_\infty^* = \frac{1}{1.12^2\pi}\left(\frac{K_{IC}}{\sigma_T}\right)^2 \approx 0.25\left(\frac{K_{IC}}{\sigma_T}\right)^2 \tag{7}$$

Equation (2) is identical to the K_{IC} criterion if $a/a^*_\infty >> 1$, which becomes:

$$\frac{\sigma_N}{\sigma_T} = \frac{\sigma_n}{\sigma_T} = \frac{1}{\sqrt{a/a_\infty^*}} \tag{8}$$

Comparing equations (5) and (8), the general asymptotic solution for small specimens can be written as [Duan and Hu, 2002, 2004a,b; Duan et al, 2004]:

$$\frac{\sigma_n}{\sigma_T} = \frac{1}{\sqrt{1 + a_e / a_\infty^*}} \tag{9}$$

Different to equations (3) and (5) that are only valid for the LEFM situation, equation (9) covers the entire fracture range from strength to K_{IC} criterion with quasi-brittle fracture in the middle.

3 ANALYSIS OF EXPERIMENTAL DATA ON SIZE EFFECT

Higgins and Bailey's experimental results [1976] are shown in Figure 2. A clear advantage of using the nominal strength σ_n is shown by the 3-p-b results. The nominal strength results of all specimens approach that of the smallest specimen (W = 5 mm) for $\alpha \approx 0$ and 1, which provides a reliable estimation of the tensile strength σ_T = 10.29 MPa.

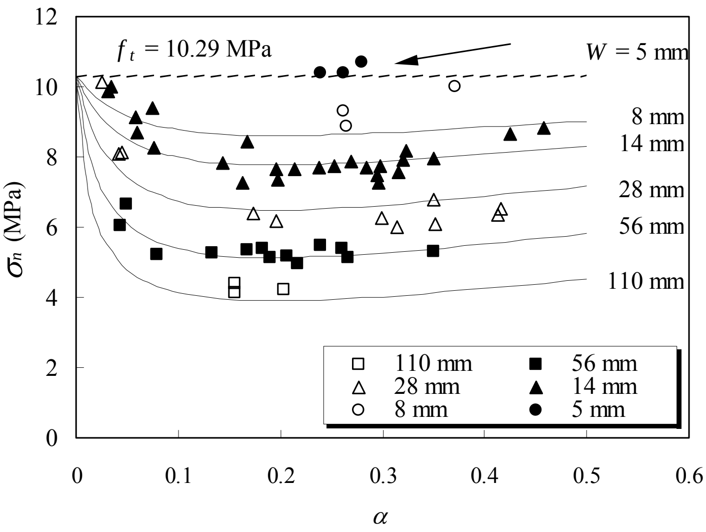

Figure 2. The comparisons of the strength predicted using the asymptotic model for finite-sized specimens with the experimental results of a hardened cement paste measured on 3-p-b specimens [Higgins and Bailey, 1976] in the system of σ_n-α.

Applications of equation (1) require geometrically similar specimens, or a constant α–ratio. Therefore, the results in Figure 2 cannot be analysed by Bažant size effect equation. However, the application of equation (9) does not require the condition of geometrically similarity. Rearranging equation (9), one can find the following linear relationship.

$$\frac{1}{\sigma_n^2} = \frac{1}{\sigma_T^2} + \frac{1}{\sigma_T^2} \cdot \frac{a_e}{a_\infty^*} \tag{10}$$

Similarly, equation (1) can be written as:

$$\frac{1}{\sigma_N^2} = \frac{1}{(A\sigma_T)^2} + \frac{1}{(A\sigma_T)^2} \cdot \frac{W}{W_*} \tag{11}$$

The results in Figure 2 are replotted in Figure 3 following the forms of equations (10) and (11). The tensile strength σ_T and fracture toughness K_{IC}

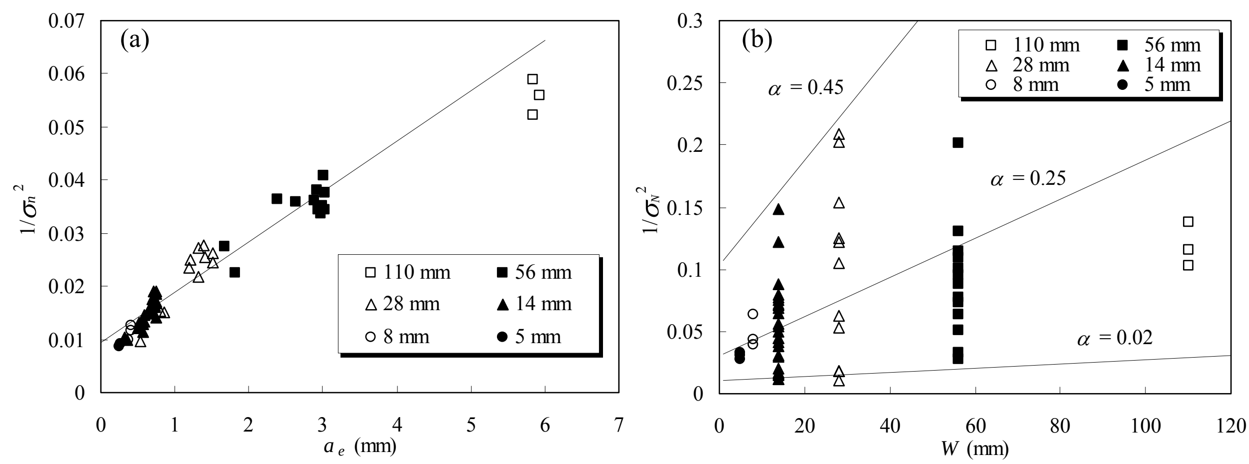

Figure 3. The comparisons of the strength predicted using the asymptotic boundary effect model with the measured data [Higgins and Bailey, 1976] in the systems of (a) $1/\sigma_n^2$ versus a_e and (b) $1/\sigma_n^2$ versus W.

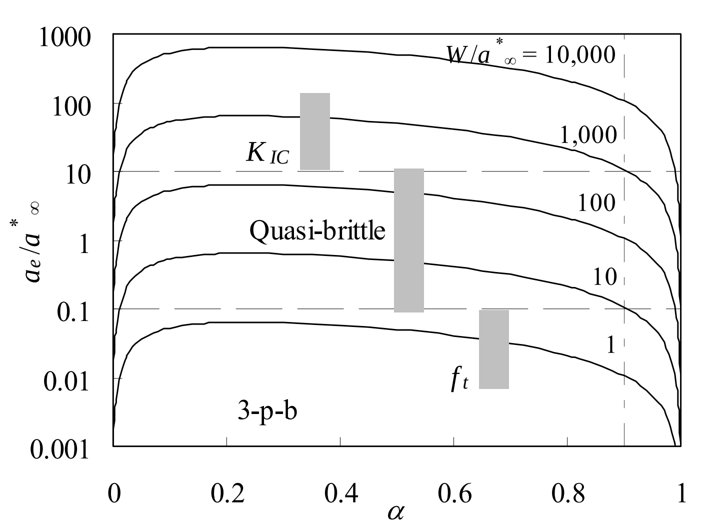

Figure 4. The changes of the equivalent crack length a_e with W and α-ratio. Fracture regions are based on the a_e/a^*_∞ ratio.

(through the reference crack a^*_∞) are determined from Figure 3(a) using equation (10). However, the nominal strength σ_N data in Figure 3(b) cannot be analysed by equation (11) because the 3-p-b specimens are not geometrically similar. Different linear relations from equation (11) for geometrically similar specimens of different α-ratios also pose some concerns. In short, different to equation (10) that provides the two important material constants σ_N and K_{IC}, equation (11) provides two curve-fitting parameters that vary with the α-ratio. The straight lines in Figure 3(b) with different α-ratios are the predictions from equation (10) based on the results in Figure 3(a).

The strength and toughness controlled fracture regions together with the transitional quasi-brittle fracture region for the most commonly used 3-p-b geometry with the span-to-depth ratio of 4 is provided in Figure 4. The crack ratio, a_e/a^*_∞, as used in equation (9) provides a convenient measurement for those different fracture regions. It is clear from Figure 4 that even very large specimens (e.g. $W/\ a^*_\infty$ > 1,000) can still experience quasi-brittle fracture or even strength controlled failure if the α-ratio is very small or close to unity showing the specimen boundary indeed influences the material fracture behaviour. To our knowledge, a clear fracture map on various fracture regions as given in Figure 4 has not been shown before.

4 DISCUSSION AND CONCLUDING REMARKS

A new asymptotic solution, equation (9), is derived for quasi-brittle fracture of concrete-like materials. The difference between the new boundary effect model and Bažant's size effect law, equation (1), is shown by the 3-p-b results as shown in Figure 3.

The present study also shows that specimen boundary indeed influences the fracture behaviour of quasi-brittle materials, which actually leads to the apparent specimen size effect.

The reference crack a^*_∞ is a material constant and a measurement of crack-tip FPZ. Therefore, comparison of its distance to the specimen boundary provide a measurement of FPZ influence on fracture behaviour. That is the reason why a/a^*_∞ and a_e/a^*_∞ play such an important role in equations (2) and (9). The equivalent crack a_e has elegantly combined contributions from both the front and back face boundaries together as shown in Figure 4.

Finally, the size effect issue dealt with by equations (1) and (9) has been studied for many years in the field of macro-fracture mechanics. It is possible some common points can be found between the macro-mechanics and the current micro-mechanics problems because micro-specimens of polysilicon have similar material-structure and specimen-size ratios.

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